ZAHRA: Research And Tought Elementary School Of Islam Journal
Volume 7, Number 2, August 2026, pp. 279-298
P-ISSN: 2723-4932 E-ISSN: 2723-7893



Tracing Students' Mathematical Reasoning: Numeracy Understanding in Contextual Addition and Subtraction Problems

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ARTICLE INFO

Article history:
Received July 27, 2026
Revised August 29, 2026
Accepted October 02, 2026
Available online November 16, 2026

Kata Kunci:
Literasi Numerasi; Pemahaman Konseptual; Operasi Penjumlahan dan Pengurangan

Keywords:
Numeracy Literacy; Conceptual Understanding; Addition and Subtraction Operations

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ABSTRAK

Penelitian ini bertujuan untuk mengkaji bagaimana siswa sekolah dasar memaknai dan menggunakan operasi penjumlahan dan pengurangan dalam menyelesaikan persoalan numerasi yang disajikan melalui konteks kehidupan sehari-hari. Penelitian ini menggunakan pendekatan kualitatif dengan desain studi kasus yang dilaksanakan pada 1–31 Agustus 2026 di Sekolah Dasar Negeri 1 Gumukrejo. Empat informan dipilih secara purposive, terdiri atas dua siswa kelas III, satu wali kelas, dan satu guru yang terlibat dalam pembelajaran matematika atau numerasi. Data dikumpulkan melalui empat kali observasi, wawancara semi-terstruktur, serta dokumentasi tugas dan hasil pekerjaan siswa. Analisis data dilakukan secara tematik melalui tahapan transkripsi, pengodean, kategorisasi, dan pengembangan tema. Kredibilitas temuan diperkuat melalui triangulasi sumber dan teknik. Hasil penelitian mengungkapkan tiga pola utama. Pertama, kelancaran prosedural siswa tidak selalu sejalan dengan pemahaman konseptual, karena siswa mampu melakukan perhitungan numerik, tetapi mengalami kesulitan dalam menentukan operasi yang tepat pada soal cerita. Kedua, konteks yang familiar membantu siswa mengenali perubahan kuantitas, tetapi tidak secara otomatis menjamin ketepatan pemilihan operasi karena interpretasi bahasa dan hubungan kuantitatif turut memengaruhi penalaran mereka. Ketiga, penjelasan lisan siswa dapat mengungkapkan pemahaman konseptual yang tidak selalu terlihat dalam jawaban tertulis, khususnya ketika siswa mampu memilih operasi yang tepat, tetapi melakukan kesalahan perhitungan. Penelitian ini memperkenalkan konsep jejak makna (meaning traces) sebagai indikator pemahaman siswa yang dapat diamati melalui kemampuan mengenali perubahan kuantitas, memilih operasi, memberikan justifikasi verbal, dan menghubungkan hasil perhitungan dengan konteks soal. Temuan ini menegaskan pentingnya mengintegrasikan penalaran konseptual dalam pembelajaran dan penilaian numerasi di sekolah dasar.

ABSTRACT

This study examines how elementary school students interpret and use addition and subtraction when solving numeracy problems presented in everyday-life contexts. Employing a qualitative approach with a case study design, the research was conducted from 1 to 31 August 2026, at State Elementary School 1 Gumukrejo. Four participants were purposively selected, comprising two third-grade students, one homeroom teacher, and one teacher involved in mathematics or numeracy instruction. Data were collected through four classroom observations, semi-structured interviews, and documentation of students' assignments and written work. The data were analyzed thematically through transcription, coding, categorization, and theme development, with credibility enhanced through source and technique triangulation. The findings reveal three patterns. First, students' procedural fluency does not necessarily correspond to conceptual understanding, as they can perform numerical calculations but struggle to identify appropriate operations in word problems. Second, familiar contexts help students recognize quantitative changes but do not automatically ensure correct operation selection, since linguistic interpretation and quantitative relationships also shape their reasoning. Third, verbal explanations reveal conceptual understanding that may not be evident in written responses, particularly when students select appropriate operations but make computational errors. This study introduces the concept of meaning traces as observable indicators of students' understanding through quantity recognition, operation selection, verbal justification, and contextual interpretation. These findings highlight the importance of integrating conceptual reasoning into elementary numeracy instruction and assessment.

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INTRODUCTION

Basic numeracy skills are often viewed as the primary indicator of student success in learning mathematics in elementary school; however, these skills do not always indicate that students understand the meaning of the arithmetic operations they use (Ebby et al., 2020; Fitriasari et al., 2025) . In learning practice, students can solve addition and subtraction problems presented in the form of symbols or number sequences relatively smoothly, but they struggle when the same problems are placed in the context of a story or a real-life situation. This situation becomes apparent when students must determine on their own whether a problem requires addition or subtraction, particularly in problems that do not directly provide clues through specific words (Odonkor & Dagadu, 2025; Lin & Riccomini, 2026).

The errors that arise are not solely caused by an inability to perform calculations, but also by limitations in understanding the relationships between pieces of information, interpreting changes in quantity, and connecting the situation at hand with the appropriate mathematical operations (Sowder, 2020; Prabowo et al., 2025). This indicates that procedural skills and numeracy understanding do not always develop in a balanced manner. Numeracy literacy requires students not only to be able to produce mathematical answers but also to understand quantitative information, select strategies, use arithmetic operations appropriately in context, and interpret the results obtained (Resnick, 2020; Tiaden, 2026). In everyday life, these types of skills are needed when children count the number of items, determine the remainder, understand changes in quantity, compare the number of things, or solve simple problems involving buying, giving, receiving, or losing (Nanda & Rani, 2025; Prabowo et al., 2025) .

Therefore, students' abilities in addition and subtraction need to be understood beyond mere accuracy in calculation results, by examining how they interpret situations, understand quantitative relationships, determine relevant operations, and construct meaning from the problem-solving process. This perspective is important for demonstrating the extent to which the learning of basic arithmetic operations has evolved from procedural mastery toward numeracy skills that can truly be applied in real-world contexts.

A number of studies indicate that students' success in performing arithmetic operations does not always align with their ability to understand mathematical problems presented in a contextual manner. **Fuson (2020)** found that elementary school students experience conceptual difficulties when solving word problems involving numbers, particularly in transforming the information in the story into an appropriate mathematical form. These difficulties demonstrate that the problem-solving process depends not only on the accuracy of

calculations but also on the ability to understand the situation and establish connections between verbal information and the mathematical concepts being used.

More specific findings were presented by McNeil et al., (2025) through a study of numeracy literacy in addition word problems. Their research results showed differences in students' abilities to understand and solve problems based on their mathematical proficiency levels; students with low proficiency tended to require assistance in solving problems, whereas students with moderate and high proficiency were able to solve them in a more structured manner. This study also underscores the importance of contextual learning in developing students' mathematical problem-solving skills.

Meanwhile, Reyna & Brainerd, (2023) identified challenges that students face when solving word problems involving addition and subtraction of whole numbers. Their findings indicate that students may succeed in one type of problem but experience difficulties when the problem context changes, particularly when they must recognize whether a situation requires addition or subtraction. This finding is consistent with Fuson (2020) and McNeil et al. (2025), who highlight the importance of understanding mathematical situations and developing appropriate strategies when solving arithmetic problems. Taken together, these studies suggest that difficulties in solving contextual problems involve more than computational accuracy. They also concern students' ability to interpret information, recognize quantitative relationships, and translate real-life situations into appropriate mathematical operations.

A second area of research concerns the relationship between procedural skills and conceptual understanding in mathematics. Procedural knowledge enables students to perform mathematical calculations, whereas conceptual understanding involves recognizing relationships between quantities and understanding why a particular operation is appropriate in a given situation (Sowder, 2020; Resnick, 2020). These two forms of knowledge are closely related, but they do not necessarily develop at the same pace. Students may perform addition and subtraction accurately when the operations are presented symbolically, yet experience difficulties when they must determine the appropriate operation independently. This distinction indicates that computational accuracy alone cannot fully represent students' understanding of arithmetic concepts. Therefore, examining how students interpret problems and justify their operational choices is necessary to obtain a more comprehensive understanding of numeracy literacy.

A third area concerns the role of language and verbal reasoning in mathematical problem-solving. Solving contextual problems requires students to understand the information presented in words, construct a representation of the situation, and connect that representation

with mathematical operations. Research by Sury and Pilchin (2025) highlights the relationship between language comprehension and applied mathematical problem-solving, indicating that students' ability to interpret numerical information is closely connected to their understanding of problem statements. In this regard, verbal explanations provide additional evidence of students' reasoning because they allow researchers to examine how students understand changes in quantity and why they select particular operations. Consequently, an incorrect written answer does not necessarily indicate a lack of conceptual understanding, as students may understand the mathematical relationship but make errors during calculation.

The novelty of this study lies in examining students' mathematical understanding through the reasoning process underlying their selection of addition and subtraction operations, rather than focusing exclusively on final calculation outcomes. Previous studies have examined students' difficulties in solving word problems, the development of arithmetic skills, and the relationship between language comprehension and mathematical problem-solving (Fuson, 2020; McNeil et al., 2025; Sury & Pilchin, 2025). However, these areas of research do not fully explain how students construct and communicate their reasoning when deciding which operation to use in a particular context. In particular, the process through which students interpret quantitative relationships, justify their operational choices, and connect these decisions with calculation outcomes requires further examination. Thus, this study addresses this gap by investigating the meaning-making processes underlying students' mathematical decisions.

To clarify the analytical focus, this study proposes a conceptual framework that connects five interrelated components of students' mathematical reasoning: understanding the problem context, identifying quantitative relationships, selecting appropriate arithmetic operations, providing verbal explanations, and producing calculation outcomes. These components represent an interconnected process rather than independent indicators. Students first interpret the situation presented in a problem and identify the relationships between the quantities involved. They then select addition or subtraction based on their understanding of those relationships, explain the reasons for their choices, and perform the required calculations. Verbal reasoning provides further insight into the conceptual understanding underlying the selected operation, while calculation outcomes indicate the accuracy of procedural execution. By examining these components together, this study seeks to distinguish conceptual understanding from procedural performance and to identify how students construct mathematical meaning when solving contextual addition and subtraction problems.

RESEARCH METHOD

This study employs a qualitative approach using a case study design to examine in depth how third-grade students interpret the use of addition and subtraction operations in the context of numeracy. The study was conducted over one month, from August 1 to 31, 2026, at Gumukrejo Public Elementary School No. 1. Four informants were selected through purposive sampling, consisting of two third-grade students, one third-grade homeroom teacher, and one teacher involved in mathematics or numeracy instruction (Abdussamad & Sik, 2021) . Two students were selected to represent a range of abilities in understanding contextual problems and determining arithmetic operations, while the two teachers were chosen because they had firsthand experience observing students' learning processes, difficulties, and strategies when tackling addition and subtraction (Sari et al., 2022; Mulyana et al., 2024).

Table 1. Characteristics of Research Informants

Informant Code	Participant	Research Role	Relevant Characteristics
S1	Third-grade student	Primary informant	Student with the ability to interpret contextual numeracy problems
S2	Third-grade student	Primary informant	Student experiencing difficulties in interpreting contextual numeracy problems
G1	Third-grade homeroom teacher	Supporting informant	Directly involved in students' classroom learning
G2	Mathematics/numeracy teacher	Supporting informant	Involved in mathematics or numeracy instruction

The table shows that the study involved two student informants as the primary sources of data and two teachers as supporting informants. Their different roles provided complementary perspectives on students' understanding and learning experiences in contextual numeracy problems. The contextual numeracy tasks focused on everyday situations involving addition and subtraction, particularly situations involving increases and decreases in quantity. The tasks were examined through five aspects: understanding the problem context, identifying quantitative relationships, recognizing changes in quantity, selecting appropriate operations, and explaining the reasons behind operational choices. These aspects provided a framework for examining students' conceptual understanding and procedural skills.

Data collection was conducted through four observation sessions, semi-structured interviews, and documentation (Kusumastuti & Khoiron, 2019). Observations focused on the students' processes of reading problems, identifying quantitative information, understanding changes in quantity, determining relevant operations, performing calculations, and explaining the meaning of the results obtained. Interviews with students were conducted individually after they completed the numeracy tasks, while interviews with teachers focused on common patterns of difficulty, the distinction between procedural skills and contextual understanding, and the learning strategies used to help students connect everyday situations with mathematical operations. The documentation included student worksheets, contextual numeracy tasks, observation notes, and interview transcripts. Student interviews further explored the reasons behind their operational choices and their understanding of changes in quantity. Teacher interviews provided supporting information about students' difficulties and learning strategies.

Data analysis was conducted thematically through the stages of transcription, repeated reading, preliminary coding, grouping codes based on semantic similarities, and theme formation. The analysis focused on the interrelationships between contextual understanding, quantitative relationships, the selection of operations, and the interpretation of results (Tisdell et al., 2025). The coding process is illustrated in Table 2 through examples of students' interview statements and their classification into codes, categories, and themes.

Table 2. Thematic Data Analysis Process

Interview Excerpts	Initial Codes	Categories	Themes
S2: "If it's just numbers, I can do the math. But with word problems, I sometimes get confused about whether to add or subtract."	Difficulty interpreting contextual problems; uncertainty in selecting operations	Contextual interpretation and operational selection	The gap between procedural skills and conceptual understanding
S1: "I chose subtraction because some of the items were given away, so the total became smaller."	Recognizing a decrease in quantity; understanding the meaning of subtraction	Understanding quantitative changes	Verbal reasoning as evidence of conceptual understanding

The table illustrates how students' interview statements were transformed into initial codes, categories, and themes. This process helped identify students' reasoning patterns and distinguish difficulties in calculation from difficulties in understanding mathematical

operations. The credibility of the findings was strengthened through triangulation of sources and techniques by comparing student data, teacher statements, observation results, and documentation (Sugiyono, 2022; Nasution & Junaidi, 2024). The specific procedures used to compare information from different participants and data collection techniques are presented in Table 3.

Table 3. Data Triangulation Procedures

Type of Triangulation	Data Sources or Techniques Compared	Purpose
Source triangulation	Statements from S1, S2, G1, and G2	To compare students' experiences and explanations with teachers' observations
Technique triangulation	Classroom observations, semi-structured interviews, and student worksheets	To examine consistency between students' verbal explanations, observed behavior, and written answers

The triangulation process enabled the researcher to examine the consistency of information across participants and data collection techniques. Comparing these sources provided a more comprehensive understanding of students' mathematical reasoning. Regarding research ethics, permission to conduct the study was obtained from the school authorities before data collection (Sugiyono, 2022; Nasution & Junaidi, 2024). Consent was also obtained from the parents or legal guardians of the student participants before their involvement in the research. The research activities were explained to the student participants in accordance with their age and level of understanding. To protect their identities, the students were identified using the codes S1 and S2, while the teachers were identified as G1 and G2. Personal information that could reveal the participants' identities was excluded from the research report, and the collected information was used solely for academic purposes.

This study has limitations because it involved only two students as the primary sources for understanding students' learning experiences and mathematical reasoning. Although two teachers provided supporting information, their perspectives did not increase the number of primary student participants. Therefore, the findings provide an in-depth account of the experiences and reasoning patterns of the two students in a specific school context. The findings are not intended to represent all elementary school students statistically. Nevertheless, the case study provides detailed insights into the relationship between

procedural skills, conceptual understanding, contextual interpretation, and verbal reasoning in solving addition and subtraction problems.

RESULT AND DISCUSSION

Result

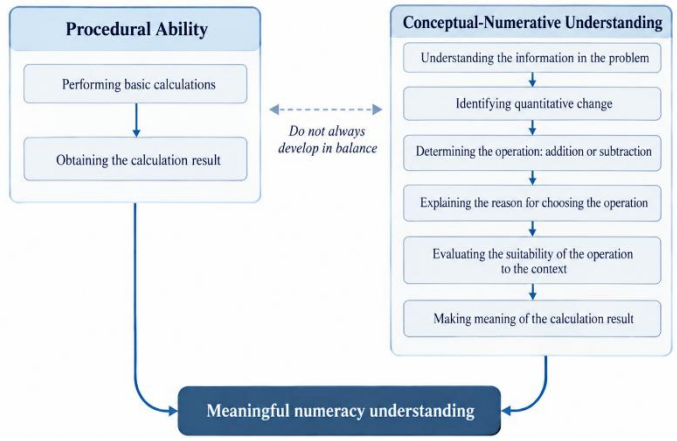
Arithmetic skills do not always indicate an understanding of the meaning of operations

The first finding indicates a gap between the ability to perform calculations and the ability to understand the rationale behind the use of a mathematical operation. In some situations, students were able to solve addition or subtraction problems presented directly in numerical form but experienced uncertainty when the same operations were embedded in story form. Difficulties primarily arise when students must determine for themselves whether a given situation involves addition or subtraction. This situation demonstrates that the accuracy of calculation results cannot always be used as a sole indicator for assessing numeracy understanding, as the process of interpreting the context plays an equally important role.

> **Teacher:** *"Between math problems and word problems, which is harder?"*
> ***S2, a third-grade student**: "If it's just numbers, I can do the math. But with word problems, I sometimes get confused about whether to add or subtract. I usually read the story first, but sometimes I still make the wrong choice."*

The results of the interview with S2 show that the main difficulty does not lie in the calculation process, but in the stage of understanding the situation presented in the problem. S2 is able to perform operations when the mathematical form is directly provided, but requires more consideration when having to determine the correct operation from a narrative. This uncertainty arises especially when the problem does not provide explicit clues regarding whether the quantity is being added to or subtracted from. S2's responses indicate that procedural skills have been developed to a certain extent, but understanding the meaning of operations still depends on the ability to read the context and connect the information in the story to the appropriate quantitative relationships. These findings confirm that computational accuracy is not always synonymous with a deep understanding of numeracy.

Findings regarding the distinction between computational skills and understanding of the meaning of operations can be mapped through stages that illustrate the processes students follow when solving numeracy problems. The indicators not only demonstrate the ability to



obtain results but also trace how students understand situations, recognize quantitative relationships, determine operations, and provide reasons for their choices. The relationships among these indicators are presented in Figure 1 to clarify the patterns of meaning-making identified in the study.

Figure 1. Indicators of the Meaning-Making Process for Addition and Subtraction Operations

These indicators show that numeracy skills go beyond students' ability to perform calculations. A more comprehensive understanding is formed when students are able to interpret situations, recognize changes in quantity, determine the relevant operation, explain the reasoning behind their choice, and interpret the results in context. Thus, the gap between procedural and conceptual skills can be identified in the stages before and after the calculation process. The more consistent the relationship between context, choice of operation, reasoning, and the meaning of the result, the stronger the students' understanding of numeracy. This pattern confirms that a correct answer is one piece of evidence of ability, but not the sole indicator of the depth of understanding.

The context of a problem influences how students construct meaning regarding addition and subtraction.

The second finding shows that the context of a problem plays a role in shaping students' understanding of addition and subtraction. The similarity of the situation in the problem to everyday experiences helps students recognize the relationship between the information presented and the required arithmetic operations. Problems that depict common activities, such as buying items, receiving additional items, or giving some items to others, tend to be easier to understand. Conversely, when information is presented through more complex stories or the relationships between numbers are not explicitly explained, students have

difficulty determining the appropriate operation even when the numbers involved are relatively simple. This situation demonstrates that understanding word problems depends not only on computational skills but also on how students interpret the presented situations. The influence of context's relevance to everyday experiences is evident in G1's explanation of students' responses when working on addition and subtraction word problems.

> **Researcher:** "Does the context of word problems affect students' understanding?"
> **G1, third-grade homeroom teacher:** *"If the story is close to the children's daily activities, they usually figure out faster whether to add or subtract. For example, problems involving buying or giving away items. But if the story is made more convoluted, they often get confused at first even though the numbers aren't difficult."*

The results of the interview with G1 show that everyday experiences help students understand the meaning of arithmetic operations in word problems. When the presented situation is familiar, students find it easier to connect the events in the story with the changes in quantity that occur. However, difficulties arise when the relationships between quantities are presented indirectly, requiring students to interpret the information before determining the operation. This indicates that a familiar context can help students understand the problem, whereas more complex story presentations demand deeper interpretive skills.

In addition to the context's relevance to everyday experiences, the structure of how information is presented in the problems also influences students' ability to understand the relationships between quantities and determine the appropriate arithmetic operation. This is reinforced by the results of interviews with G2, which revealed differences in students' responses when faced with problems featuring more complex information presentation.

> **Researcher:** "How do students respond if the story format in the problem is changed?"
> **G2, 3rd-grade math teacher:** *"Usually, if the story is simple, the children understand more quickly. But if the sentences are changed or the information isn't presented directly, they start to get confused. Some immediately add the numbers because they see two figures, even though they should actually subtract. So, sometimes they haven't yet grasped the meaning of the story."*

G2's explanation illustrates that students' difficulties are not only related to the type of operation used but also to their ability to understand the relationships between quantities in the story. When information is presented indirectly, students may determine the operation based on the visible numbers without first understanding the changes in quantity that are taking place. This situation demonstrates that familiarity with numbers alone is insufficient to help students solve word problems if it is not accompanied by the ability to interpret the

underlying mathematical relationships. Thus, changes in the story's structure can pose a challenge for students even if the level of computational difficulty remains the same.

Both interviews indicate that the context of the problem and the way it is presented play a crucial role in building students' understanding of addition and subtraction. G1 emphasized that situations closely related to everyday experiences help students recognize the meaning of operations, while G2 described the difficulties that arise when information in a problem is presented indirectly. These findings show that numeracy understanding is determined not only by the ability to perform calculations but also by the ability to connect information in a story with the changes in quantity that occur. Therefore, the use of diverse contexts in learning is necessary to help students develop an understanding that is not limited to familiar situations but can also be applied to problems presented in different ways.

Students' verbal explanations reveal insights not evident from their written answers.

The third finding indicates that errors in solving addition and subtraction problems do not always reflect students' low understanding of the meaning of arithmetic operations. In some cases, students provided written answers that were not entirely accurate but were able to explain their choice of operation based on the situation presented in the problem. This was evident when students could recognize an increase or decrease in quantity even while experiencing difficulty in performing the calculations. These verbal explanations demonstrated a conceptual understanding that was not fully captured by their written work.

In subtraction problems, S1 was able to explain the reason for using the operation even though the calculation result was incorrect. This is illustrated in the following interview excerpt.

> **Teacher:** *"Why did you choose subtraction, even though your calculation was incorrect?"*
> **Student S1, third grader:** *"I chose subtraction because some of the items were given away, so the total became smaller. My calculation was wrong just now, but I know the final total has to be less."*

The results of the interview with S1 show that a calculation error does not necessarily mean the student has not understood the concept of subtraction. The explanation provided demonstrates that S1 is able to recognize a change in quantity when some items are given to others. The student understood that this action caused the initial quantity to decrease, making subtraction the appropriate operation. Although the calculation result was not yet accurate, the reasoning provided indicates that the student has understood the relationship between the situation in the problem and the meaning of subtraction. Thus, the difficulty experienced by

S1 was more evident in the execution of the calculation than in determining the arithmetic operation.

A similar understanding is also evident in addition. Although the calculation results are not yet accurate, the student can explain that addition is used because there is an increase in quantity in the described situation. This is illustrated through the following dialogue example.

> **Teacher:** *"Why did you choose addition, even though your answer was wrong?"*
> **Student S2, a third-grader:** *"Because there were already some items to begin with, and then I got more, so the total increased. That's why I added them, but I made a mistake while calculating."*

Student S2's explanation shows that errors in calculation results are not always caused by an inability to understand the meaning of addition. The student was able to recognize that the addition of more items caused the total quantity to increase, making addition the appropriate operation. Although the final answer was incorrect, the reasoning provided demonstrates that the student understood the relationship between an increase in quantity and the use of the addition operation. The error that occurred was more related to the counting process, whereas the choice of operation was based on an understanding of the situation presented in the problem.

To provide a more systematic account of these findings, Table 1 presents an analysis of the subtraction and addition cases based on students' written responses, selected operations, calculation results, and verbal explanations.

Table 4. Empirical Analysis of Students' Reasoning in Contextual Addition and Subtraction Problems

Analytical Aspects	Subtraction Case (S1)	Addition Case (S2)
Problem Context	A situation involving giving away some items, resulting in a decrease in quantity.	A situation involving receiving additional items, resulting in an increase in quantity.
Students' Written Responses	The written calculation was incorrect.	The written calculation was incorrect.
Appropriateness of the Chosen Operations	Appropriate. The student correctly selected subtraction because the quantity decreased.	Appropriate. The student correctly selected addition because the quantity increased.
Calculation Results	Incorrect calculation result.	Incorrect calculation result.
Verbal Justifications	The student explained that some items were given away, causing the total quantity to decrease.	The student explained that receiving additional items caused the total quantity to increase.
Researcher's Interpretations	The student demonstrated an understanding of decreasing	The student demonstrated an understanding of increasing

	quantity and the meaning of subtraction, although the calculation procedure was not yet accurate.	quantity and the meaning of addition, although the calculation procedure was not yet accurate.

The analysis in Table 1 demonstrates that both students were able to select appropriate arithmetic operations based on their understanding of changes in quantity, although their calculation results were incorrect. Their verbal explanations provided evidence of conceptual understanding that was not fully reflected in their written answers. These findings indicate that students' mathematical understanding should be examined through both their calculation performance and their explanations of the operations they select.

Both interviews showed that written answers do not always fully reflect students' understanding. In subtraction, Student 1 recognized that the total decreases when some items are given away, while in addition, Student 2 understood that the total increases when additional items are received. Although both produced calculations that were not entirely accurate, their verbal explanations showed that they were able to connect the situation in the problem with the appropriate meaning of the operation. These findings confirm that procedural errors need to be distinguished from conceptual difficulties when assessing students' numeracy skills. Through verbal explanations, teachers can gain a clearer picture of students' thought processes so that assessment is not solely based on the accuracy of the final answer but also considers the understanding underlying the choice of arithmetic operation.

To clarify the third finding, the following visualization is presented to illustrate the difference between the information apparent in written answers and the understanding revealed through students' verbal explanations. Figure 2 illustrates that an inaccurate final result does not necessarily mean that a student has failed to understand the concept underlying the problem. Through the reasons given, students' thought processes can be traced in greater depth, particularly in recognizing changes in quantity, selecting operations, and explaining the mathematical meaning of the situation at hand.

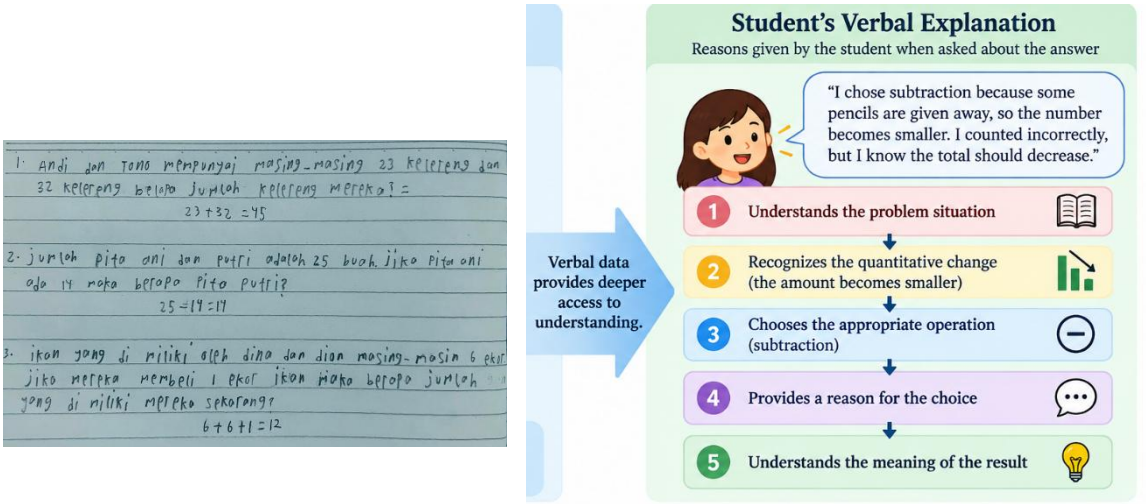


Figure 2. Verbal explanations reveal an understanding that goes beyond written answers.

As shown in Figure 2, written answers provide only a superficial picture of student performance, whereas verbal explanations offer deeper insight into the structure of their understanding. This illustration shows that errors can occur at the procedural level, even though students have understood that the situation in the problem represents subtraction. Therefore, verbal serve as *meaning-making traces* that help distinguish between calculation errors and conceptual understanding that has already been formed.

Discussion

Calculation skills do not always indicate an understanding of the meaning of operations.

The finding that computational ability does not always indicate an understanding of the meaning of operations confirms the distinction between procedural knowledge and conceptual knowledge in students' numeracy development. Procedural skills enable students to perform calculation steps, whereas conceptual understanding requires the ability to recognize relationships between numbers and explain why a particular operation is relevant to a specific situation. The responses of S1 and S2 demonstrate that these two dimensions are interrelated but do not necessarily develop at the same pace. Both students were able to identify the appropriate operations based on changes in quantity, although their calculation results were incorrect. Meanwhile, S2's explanation in the first finding also indicates that the ability to perform calculations using direct numerical representations does not automatically guarantee the ability to interpret operations in word problems (Rinne et al., 2020) .

These findings are consistent with Demirci & Ergül, (2026), who show that difficulties in solving additive word problems are related to conceptual understanding and cannot be

explained solely by students' ability to execute calculation procedures. Gesuelli & Jordan, (2024), also highlight the importance of recognizing mathematical structures and relationships in developing meaningful mathematical understanding. Similarly, Sury & Pilchin, (2025) demonstrate that understanding subtraction as a relationship or difference requires students to recognize relationships between quantities rather than merely produce numerical answers.

The present study affirms previous research on the relationship between conceptual and procedural knowledge. However, it extends this understanding by showing how differences between these dimensions become visible when third-grade students translate everyday situations into arithmetic operations. The cases of S1 and S2 demonstrate that students may understand why addition or subtraction is appropriate while still making errors during calculation. This distinction suggests that procedural performance and conceptual understanding should be examined as related but separate dimensions of numeracy competence. Therefore, assessing students solely through the accuracy of their final answers may overlook important evidence of their conceptual understanding (Porter, 2025).

The context of the problem influences how students construct meaning regarding addition and subtraction.

The finding that the context of a problem influences how students construct meaning regarding addition and subtraction can be understood through the perspective of numeracy literacy, which views mathematics as a process of connecting real-world situations with mathematical representations and actions. Before determining the operation, students need to construct mental representations of what is happening, how quantities change, and what relationships are formed among the information in the problem (Chinn, 2020). This finding aligns with research on the role of extra-mathematical knowledge, which shows that understanding real-life contexts interacts with mathematical ability when students solve word problems.

Studies on *realistic word problems* also show that students may produce incorrect responses when they ignore or misapply real-world knowledge in interpreting problems (Waywood,2021). However, a meta-analysis by Nur et al. (2022) offers a more critical perspective: a familiar situational context alone does not consistently improve performance on word problems; linguistic structure, the position of unknown information, and the need for realistic consideration also play a role. Thus, the findings of this study reinforce the importance of context, but at the same time refine previous theories by showing that familiarity is not an automatic factor. For the third-grade students in this study, a context

close to their own experiences appears to serve as a bridge for recognizing changes in quantity, while ultimate success still depends on the ability to interpret the mathematical structure underlying the story.

Students' verbal reasoning reveals understanding not evident in written answers.

The finding that students' verbal justifications can reveal understanding not apparent in written answers reinforces the view that numeracy literacy involves representation and reasoning rather than merely producing numerical answers. Solving word problems requires students to construct a mental representation of a situation, translate it into a mathematical model, and perform the required calculation. Since difficulties may occur at different stages of this process, incorrect final answers do not always indicate conceptual failure. Research on the demands of word problems also identifies the construction of situational and mathematical models as an important stage before calculation begins (Birgin et al., 2022).

These findings are consistent with Green, (2020), who demonstrates that visualizing relationships between pieces of information can help students use conceptual knowledge when solving contextual arithmetic problems. Furthermore, research on elementary school students indicates that word-problem-solving abilities are associated with language resources that are distinct from direct arithmetic skills. Therefore, numerical performance alone is insufficient to represent students' overall mathematical ability (Clements, 2022).

The present study extends these findings by developing the concept of "traces of meaning" as evidence of students' conceptual understanding during contextual numeracy problem-solving. In this study, traces of meaning refer to observable indicators in students' reasoning that reveal how they interpret mathematical situations and connect them with appropriate operations. These indicators consist of four interrelated aspects: (1) recognizing changes in quantity, (2) selecting appropriate arithmetic operations, (3) providing verbal justifications for their operational choices, and (4) connecting calculation results back to the original problem context.

The cases of S1 and S2 illustrate these indicators. S1 recognized that giving away some items caused the total quantity to decrease and therefore selected subtraction. Similarly, S2 recognized that receiving additional items increased the total quantity and therefore selected addition. Both students were able to explain their choices even though their calculation results were incorrect. These explanations provide evidence that their understanding of the meaning of operations could be identified through their reasoning rather than solely through their written answers.

This contribution affirms previous research on the importance of situational representation, mathematical reasoning, and language in solving word problems. However, it extends those perspectives by organizing students' reasoning into four indicators that can be used to examine the meaning-making process in addition and subtraction. Furthermore, this study refines the tendency to evaluate numeracy understanding primarily through correct or incorrect answers by positioning students' verbal justifications and connections between context and mathematical results as additional sources of evidence. Thus, "traces of meaning" offer an interpretive perspective for distinguishing procedural errors from conceptual understanding and for examining how students construct mathematical meaning when solving contextual problems.

CONCLUSION

This study demonstrates that numeracy literacy in addition and subtraction cannot be understood solely through the accuracy of calculation results but must be examined in terms of how students interpret context, recognize quantitative relationships, select operations, and explain the mathematical reasoning behind their decisions. The findings reveal that procedural skills can develop without being accompanied by equivalent conceptual understanding; that contexts closely related to students' experiences can serve as a bridge to meaning but do not automatically guarantee success; and that verbal explanations can reveal structures of understanding that are not always apparent in written answers.

Theoretically, this study reinforces the relationship between procedural knowledge, conceptual understanding, situational representations, and reasoning in numeracy literacy, while expanding upon it by positioning the process of selecting operations and verbal reasoning as "traces of meaning" that can be used to gauge the depth of students' understanding. In this study, "traces of meaning" represent observable indicators of students' conceptual understanding through their ability to recognize changes in quantity, select appropriate operations, provide verbal justifications, and connect calculation results with the original problem context. This concept contributes an alternative perspective to numeracy assessment by emphasizing the reasoning processes underlying students' mathematical decisions.

Practically, these findings suggest that teachers should provide contextual word problems with varied linguistic structures to help students develop the ability to interpret different forms of mathematical information. Teachers should also encourage students to explain and justify their choices of arithmetic operations before and after performing

calculations. Furthermore, assessment should distinguish between calculation errors and conceptual misunderstandings so that students' learning difficulties can be identified more accurately. These practices can help teachers evaluate not only the answers students produce but also the reasoning underlying their mathematical decisions.

This study has several limitations. First, it involved four informants, consisting of only two third-grade students as the primary sources of information and two teachers as supporting informants. Second, the research was conducted at only one elementary school, Gumukrejo Public Elementary School No. 1, which limits the transferability of the findings to other educational contexts. Third, the study focused exclusively on addition and subtraction operations and therefore does not address students' reasoning in other mathematical operations.

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